

## Problem Set: Analysis I

1. Solve the following equations.

(a)  $x^2 - 6x + 8 = 0$

(b)  $(3x - 1)^2 - (5x - 3)^2 = -(4x - 2)^2$

(c)  $\sqrt{x^2 - 9} = 9 - x$

(d)  $\log_x(2x + 8) = 2$

(e)  $e^{2x-5} + 1 = 4$

(f)  $\log_2 \frac{2}{x} = 3 + \log_2 x$ , where  $x > 0$

(g)  $(27)^{2x+1} = \frac{1}{3}$

2. Simplify the following expressions.

(a)  $\frac{4^2 \cdot 6^2}{3^3 \cdot 2^3}$

(b)  $\frac{(x+1)^3(x+1)^{-2}}{(x+1)^2(x+1)^{-3}}$

(c)  $(-3xy^2)^3$

(d)  $\frac{\frac{(x^2)^3}{x^4}}{\left(\frac{x^3}{(x^3)^2}\right)^{-2}}$

(e)  $[(2x+1)(2x-1)](4x^2+1)$

(f)  $\frac{6x^5 + 4x^3 - 1}{2x^2}$

(g)  $\frac{1 + 4x^2 + 6x}{2x - 1}$

(h)  $\frac{x^2 - 5x + 4}{x^2 + 2x - 3} - \frac{x^2 + 2x}{x^2 + 5x + 6}$

3. Show that:

(a)  $\sum_{i=1}^N (x_i - \mu_x)^2 = \sum_{i=1}^N x_i^2 - N\mu_x^2$ . Hint: Note that  $\mu_x = \frac{1}{N} \sum_{i=1}^N x_i$ .

(b)  $\sum_{i=1}^n (a_{i+1} - a_i) = a_{n+1} - a_1$ .

4. Show that  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ .

5. Differentiate the following functions with respect to  $x$ .

(a)  $f(x) = 7x^3 - 2x^2 - 5x + 1$

(b)  $f(x) = 0.7x^{-4} + 1.3 - 3.1x^3$

(c)  $f(x) = \frac{3x^2 + 1}{2x}$

(d)  $f(x) = \sqrt{4x + 9}$

(e)  $f(x) = \frac{x^{\frac{1}{3}} - 2}{(x^5 - 2)^3}$

(f)  $f(x) = \ln \left( \frac{x^2}{x^4 + 1} \right)$

(g)  $f(x) = e^{x^3 + x}$

(h)  $f(x) = \frac{1}{e^x + e^{-x}}$

6. Find the all first and second (mixed) partial derivatives of the following functions.

(a)  $f(x, y) = \ln x \cdot y^2$

(b)  $f(x, y) = \sqrt{2x - y}$

(c)  $f(x, y) = (x + 4y)(e^{-2x} + e^{-3y})$

7. For what value of  $a$  is the following function continuous for all  $x$ ? Is it also differentiable for all  $x$  for this value of  $a$ ?

$$f(x) = \begin{cases} ax - 1 & \text{if } x \leq 1 \\ 3x^2 + 1 & \text{if } x > 1 \end{cases}$$